

QED collinear radiation factors in the next-to-leading logarithmic approximation

A.B. Arbuzov, E.S. Scherbakova

*Bogoliubov Laboratory of Theoretical Physics,
JINR, Dubna, 141980 Russia*

Abstract

The effect of the collinear photon radiation by charged particles is considered in the second order of the perturbation theory. Double and single photon radiation is evaluated. The corresponding radiation factors are obtained. The QED renormalization group approach is exploited in the next-to-leading order. The results are suited to perform a systematic treatment of the second order next-to-leading logarithmic radiative corrections to various processes either analytically or numerically.

Key words: QED radiative corrections, next-to-leading approximation

PACS: 13.40.-f Electromagnetic processes and properties 12.15.Lk Electroweak radiative corrections

1 Introduction

The modern high energy physics experiments with advanced techniques and high statistical require adequately precise theoretical predictions. Among various effects which have to be taken into account, QED radiative corrections (RC) give important contributions to the predictions. At high energies they are usually computed with help of the QED perturbation theory. But direct computations of higher order QED corrections to complicated processes can be rather cumbersome. For this reason certain methods were developed to evaluate first the numerically most important contributions. In particular, besides the expansion in the powers of the fine structure constant α , one can use an expansion in powers of the so-called large logarithm, $L = \ln(M^2/m^2)$, where M is a large energy scale, and m is a charged particle mass, $m \ll M$.

In this paper we present the derivation of a particular contribution of QED RC of the order $\mathcal{O}(\alpha^2 L^{2,1})$. It is well known, that the angular distribution of a photon emitted by a high-energy particle is peaked in the forward direction. Moreover, it is easy to show starting from the matrix element, that a process with emission of a collinear photon can be represented in a factorized form (see *e.g.* Ref. [1]). As usually the factorization appear if it is possible to separate the long-distance sub-process of collinear photon emission and the main short-distance sub-process. In other words, we assume that the experimental conditions of the particle registration allow to neglect the effects of small

changes of transverse momenta arising from emission of the photon at the small angle with respect to its parent particle: $\vartheta_\gamma \ll 1$. So the cross section (or the decay width) of the process with hard collinear photon emission can be represented as a convolution of the radiation factor R and the distribution of the radiation-less processes $d\hat{\sigma}$ (in example of the $2 \rightarrow 2$ type):

$$\begin{aligned} d\sigma[a(p_1) + b(p_2) \rightarrow c(q_1) + d(q_2) + \gamma(k \sim (1-z)p_1)] \\ = d\hat{\sigma}[a(zp_1) + b(p_2) \rightarrow c(q_1) + d(q_2)] \otimes R_H^{\text{ISR}}(z), \\ d\sigma[a(p_1) + b(p_2) \rightarrow c(q_1) + d(q_2) + \gamma(k \sim (1-z)q_1)] \\ = d\hat{\sigma}[a(p_1) + b(p_2) \rightarrow c(q_1) + d(q_2)] R_H^{\text{FSR}}(z), \end{aligned} \quad (1)$$

where $z = E'/E$ is the energy fraction of the particle emitted the photon, E and E' are the charged particle energy *before* and *after* radiation of the photon, respectively. In the case of the final state radiation (FSR), we observed the energy of particle c being equal to zq_1^0 , and we have a direct product of the two factors. In the case of the initial state radiation (ISR), we usually compute the kernel cross section in the center-of-mass reference frame of particles $a(zp_1)$ and $b(p_2)$ and then perform a relativistic boost to the laboratory reference frame.

This paper is organized as follows. In the next Section we re-call the known results for the first order collinear radiation factors. In Sect. 3 and in Sect. 4 we present our results for the second order radiation factors for double and single hard photon radiation, respectively. Possible applications of the results are discussed in Conclusions. In Appendix we give the explicit formulae for QED splitting functions used in the derivation of the factors.

2 The First Order Approximation

The derivation of the collinear radiation factors due to an emission of a single hard photon in $\mathcal{O}(\alpha)$ can be found in Ref. [1]. The factors read

$$R_H^{\text{ISR}}(z) = \frac{\alpha}{2\pi} \left[\frac{1+z^2}{1-z} \left(\ln \frac{E^2}{m^2} - 1 + l_0 \right) + 1 - z + \mathcal{O}\left(\frac{m^2}{E^2}\right) + \mathcal{O}(\vartheta_0^2) \right], \quad (2)$$

$$R_H^{\text{FSR}}(z) = \frac{\alpha}{2\pi} \left[\frac{1+z^2}{1-z} \left(\ln \frac{E^2}{m^2} - 1 + l_0 + 2 \ln z \right) + 1 - z + \mathcal{O}\left(\frac{m^2}{E^2}\right) + \mathcal{O}(\vartheta_0^2) \right]. \quad (3)$$

The mass of the particle m is assumed to be small compared with the energy, and terms suppressed by the factor m^2/E^2 are omitted. The photon emission angle with respect to its parent particle is restricted by the condition

$$\vartheta_\gamma < \vartheta_0, \quad \frac{m}{E} \ll \vartheta_0 \ll 1, \quad l_0 = \ln \frac{\vartheta_0^2}{4}. \quad (4)$$

The energy of the emitted photon is assumed to be above a certain threshold, $E_\gamma > \Delta E$. The parameters ϑ_0 and Δ either might be related to concrete experimental conditions, or serve as auxiliary quantities. In the latter case they should cancel out after summing

up the contributions due to emission of the collinear hard photons with the ones of non-collinear hard photons and of soft photons. These $\mathcal{O}(\alpha)$ radiation factors are universal and describe collinear single photon emission for various high-energy processes [1].

3 Double Hard Photon Radiation

In paper [2] the effect of the double hard photon radiation in the Bhabha scattering was considered. In particular the effect of the two photon emission inside a collinear cone along the direction of motion of any of the 4 charged particles in this process was presented in a form being differential in the energy fraction of both the photons. So to get the collinear radiation factor, we have just to integrate over one of the energy fractions keeping their sum fixed. The lower limit of the integral over the photon energy fraction is chosen to be equal to the parameter Δ because both the photons should be hard and have therefore energy above ΔE . In this way for the case of the initial state radiation we got

$$\begin{aligned} R_{\text{HH}}^{\text{ISR}}(z) = & \left(\frac{\alpha}{2\pi}\right)^2 L \left\{ (L + 2l_0) \left(\frac{1+z^2}{1-z} (2\ln(1-z) - 2\ln\Delta - \ln z) + \frac{1+z}{2} \ln z \right. \right. \\ & \left. \left. - 1 + z \right) + \frac{1+z^2}{1-z} \left(\ln^2 z + 2\ln z - 4\ln(1-z) + 4\ln\Delta \right) \right. \\ & \left. + (1-z) \left(2\ln(1-z) - 2\ln\Delta - \ln z + 3 \right) + \frac{1+z}{2} \ln^2 z \right\}, \end{aligned} \quad (5)$$

where z is, as in Eq. (2), the energy fraction of the charged particle *after* the emission of the two photons.

The corresponding radiation factor for the final state radiation case can be obtained from the ISR one by means of the Gribov–Lipatov relation:

$$\begin{aligned} R_{\text{HH}}^{\text{FSR}}(z) = & -z R_{\text{HH}}^{\text{ISR}}\left(\frac{1}{z}\right) \Big|_{\ln\Delta \rightarrow \ln\Delta - \ln z; \ l_0 \rightarrow l_0 + 2\ln z} = \left(\frac{\alpha}{2\pi}\right)^2 L \left\{ (L + 2l_0) \left[\frac{1+z^2}{1-z} \right. \right. \\ & \times \left(2\ln(1-z) - 2\ln\Delta + \ln z \right) + \frac{1+z}{2} \ln z - 1 + z \Big] \\ & + \frac{1+z^2}{1-z} \left(5\ln^2 z - 2\ln z - 4\ln(1-z) + 4\ln\Delta + 8\ln z (\ln(1-z) - \ln\Delta) \right) \\ & \left. + (1-z) \left(2\ln(1-z) - 2\ln\Delta - 3\ln z + 3 \right) + \frac{3(1+z)}{2} \ln^2 z \right\}, \end{aligned} \quad (6)$$

Note that the additional interchanges in the above relation applied for $\ln\Delta$ and l_0 appear in our case from the crossing relations of the two channels with the given cuts on the energies of the soft photon and on the photon emission angle.

4 Single Hard Photon Radiation

We have to consider also the process of single hard photon emission accompanied by the one-loop virtual correction or by the emission of a soft photon. As concerns soft

photon radiation, its contribution does factorize with respect to the collinear hard photon emission:

$$d\sigma_{\text{HS}} = R_{\text{H}} \otimes \delta_{\text{S}} d\sigma^{(0)}, \quad \delta_{\text{S}} = \frac{d\sigma_{\text{Soft}}^{(1)}}{d\sigma^{(0)}}, \quad (7)$$

where δ_{S} is the one-loop soft photon radiation factor for the given process, computed in the standard way [3]. This quantity has an infra-red divergence, which cancels out after summation with the virtual loop contribution. And $d\sigma^{(0)}$ is the Born level cross section.

So we would like to get the radiation factor $R_{\text{H}(\text{S}+\text{V})}^{\text{ISR}}(z)$, where both the one-loop virtual correction and soft photon radiation are taken into account. To find this radiation factor, we will exploit the known result for the complete second order NLO QED corrections provided by the renormalization group approach [4,5,6,7,8]. In analogy to QCD we can write the master formula for the corrected cross section *e.g.* for Bhabha scattering in the form (see Ref. [9]):

$$\begin{aligned} d\sigma = & \int_{\bar{z}_1}^1 dz_1 \int_{\bar{z}_2}^1 dz_2 \mathcal{D}_{ee}^{\text{str}}(z_1) \mathcal{D}_{ee}^{\text{str}}(z_2) \left(d\sigma^{(0)}(z_1, z_2) + d\bar{\sigma}^{(1)}(z_1, z_2) + \mathcal{O}(\alpha^2 L^0) \right) \\ & \times \int_{\bar{y}_1}^1 \frac{dy_1}{Y_1} \int_{\bar{y}_2}^1 \frac{dy_2}{Y_2} \mathcal{D}_{ee}^{\text{frg}}\left(\frac{y_1}{Y_1}\right) \mathcal{D}_{ee}^{\text{frg}}\left(\frac{y_2}{Y_2}\right), \end{aligned} \quad (8)$$

where $d\bar{\sigma}^{(1)}$ is the $\mathcal{O}(\alpha)$ correction to the massless scattering, calculated using the $\overline{\text{MS}}$ scheme to subtract the mass singularities. The energy fractions of the incoming partons are $z_{1,2}$, and $Y_{1,2}$ are the energy fractions of the outgoing electron and positron. $\mathcal{D}_{ee}^{\text{str}(\text{frg})}$ are the structure (fragmentation) functions of an electron. Here we consider only the photonic contributions to the non-singlet part of these functions. The radiation factors corresponding to the collinear emission of light pairs were evaluated in Ref. [10]. With help of the master formula we can find the most important contributions reinforced by the large logarithm L in radiative corrections to a wide class of other processes as well.

We are going to drop the pair contributions, so we need here the pure photonic part of the non-singlet structure (fragmentation) functions for the initial (final) state corrections. These functions describe the probability to find a massless (massive) electron with energy fraction z in the given massive (massless) electron. In our case with the next-to-leading accuracy we have

$$\begin{aligned} \mathcal{D}_{ee}^{\text{str,frg}}(z) = & \delta(1-z) + \frac{\alpha}{2\pi} d^{(1)}(z, \mu_0, m_e) + \frac{\alpha}{2\pi} L P^{(0)}(z) \\ & + \left(\frac{\alpha}{2\pi} \right)^2 \left(\frac{1}{2} L^2 P^{(0)} \otimes P^{(0)}(z) + L P^{(0)} \otimes d^{(1)}(z, \mu_0, m_e) \right. \\ & \left. + L P_{ee}^{(1,\gamma)\text{str,frg}}(z) \right) + \mathcal{O}(\alpha^2 L^0, \alpha^3). \end{aligned} \quad (9)$$

The difference between the functions appear only due to the difference in the next-to-order splitting functions $P^{(1,\gamma)}$, given in the Appendix together with the other relevant

functions. The modified minimal subtraction scheme $\overline{\text{MS}}$ is used. We have chosen the factorization scale equal to E , and the renormalization scale μ_0 will be taken equal to m_e . More details on the application of the approach to calculation of second order next-to-leading QED corrections can be found in Refs. [5,8].

Let us consider the $\mathcal{O}(\alpha^2 L^n)$ ($n > 0$) radiative corrections to a given process, which are related to at least one hard photon emission. They can be separated into four parts according to their kinematics:

$$\delta_{\text{Hard}}^{(2)\text{NLO}} = \delta_{\text{HH(coll)}}^{(2)} + \delta_{\text{HH(s-coll)}}^{(2)} + \delta_{(\text{S+V})\text{H(n-coll)}}^{(2)} + \delta_{(\text{S+V})\text{H(coll)}}^{(2)}, \quad (10)$$

where $\delta_{\text{HH(coll)}}^{(2)}$ gives the contribution of double hard photon emission considered in the previous section. The case when one of the photons is emitted at large angle ($\vartheta_\gamma > \vartheta_0$) and the other one is collinear is denoted $\delta_{\text{HH(s-coll)}}^{(2)}$, where “s-coll” means a semi-collinear kinematics, see Ref. [2] for details. The term $\delta_{(\text{S+V})\text{H(n-coll)}}^{(2)}$ corresponds to single hard non-collinear ($\vartheta_\gamma > \vartheta_0$) photon emission accompanied by the $\mathcal{O}(\alpha)$ soft and virtual photonic corrections. Note that since the non-collinear photon emission doesn't give rise to the large logarithm, we can keep in $\delta_{(\text{S+V})\text{H(n-coll)}}^{(2)}$ only the leading logarithmic terms in the sum of soft and virtual corrections. And the last term is the contribution that we are looking for: the one due to single hard collinear photon emission accompanied by $\mathcal{O}(\alpha)$ soft and virtual corrections.

From the other hand, the same quantity can be found in the master formula (8):

$$\begin{aligned} \delta_{\text{Hard}}^{(2)\text{NLO}} = & \frac{\alpha}{2\pi} LP_{\Theta}^{(0)} \otimes d\bar{\sigma}_{\Theta}^{(1)} + \frac{\alpha}{2\pi} LP_{\Delta}^{(0)} \otimes d\bar{\sigma}_{\Theta}^{(1)} + \frac{\alpha}{2\pi} (LP_{\Theta}^{(0)} + d_{\Theta}^{(1)}) \otimes d\bar{\sigma}_{\Delta}^{(1)} \\ & + \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{L^2}{2} P^{(0)} \otimes P^{(0)} + LP^{(0)} \otimes d^{(1)} + LP^{(1,\gamma)\text{str}}\right)_{\Theta} \otimes d\sigma^{(0)}, \end{aligned} \quad (11)$$

where we leaved out the splitting functions arguments for short. Here $d\bar{\sigma}_{\Theta}^{(1)}$ is the contribution of single hard photon emission and $d\bar{\sigma}_{\Delta}^{(1)}$ is the soft-virtual contribution (in the $\overline{\text{MS}}$ scheme with massless electrons). Lower indexes Θ and Δ mean here the parts of the corresponding functions related to hard and soft plus virtual radiation, respectively. Again we kept in the above equation only the terms reinforced by the large logarithm L .

Comparing the two expression (10) and (11) we get

$$\begin{aligned} \delta_{(\text{S+V})\text{H(coll)}}^{(2)} = & R_{\text{H(S+V)}}^{\text{ISR}}(z) \otimes d\hat{\sigma}(z) = \frac{\alpha}{2\pi} LP^{(0)} \otimes d\bar{\sigma}_{\Theta}^{(1)} + \frac{\alpha}{2\pi} LP_{\Theta}^{(0)} \otimes d\bar{\sigma}_{\Delta}^{(1)} \\ & + \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{L^2}{2} P^{(0)} \otimes P^{(0)} + LP^{(0)} \otimes d^{(1)} + LP^{(1,\gamma)\text{str}}\right)_{\Theta} \otimes d\sigma^{(0)} \\ & - \frac{\alpha}{2\pi} R_{\Theta}^{(0)} \otimes d\sigma_{\Theta}^{(1)} \Big|_{\vartheta_\gamma \geq \vartheta_0} - \frac{\alpha}{2\pi} LP_{\Delta}^{(0)} \otimes d\sigma_{\Theta}^{(1)} \Big|_{\vartheta_\gamma \geq \vartheta_0} - \left(\frac{\alpha}{2\pi}\right)^2 R_{\text{HH}}^{\text{ISR}} \otimes d\sigma^{(0)}. \end{aligned} \quad (12)$$

The $\overline{\text{MS}}$ subtraction leads to the following relations:

$$d\bar{\sigma}_{\Delta}^{(1)} = d\sigma_{\text{Soft}}^{(1)} + d\sigma_{\text{Virt}}^{(1)} - \frac{\alpha}{2\pi} (LP_{\Delta}^{(0)} + d_{\Delta}^{(1)}) d\sigma^{(0)}, \quad (13)$$

$$d\bar{\sigma}_\Theta^{(1)} = d\sigma_\Theta^{(1)} - \frac{\alpha}{2\pi}(LP_\Theta^{(0)} + d_\Theta^{(1)}) \otimes d\sigma^{(0)}. \quad (14)$$

Summing up the parts in (12) proportional to $\sigma_\Theta^{(1)}$ with help of (14) we arrive at

$$\frac{\alpha}{2\pi}LP_\Theta^{(0)} \otimes d\sigma_\Theta^{(1)} \Big|_{\vartheta_\gamma < \vartheta_0} = \frac{\alpha}{2\pi}LP_\Theta^{(0)} \otimes \frac{\alpha}{2\pi}R_H^{\text{ISR}} \otimes d\sigma^{(0)}. \quad (15)$$

After substitution (13) and (15) to (12) we get the result

$$\begin{aligned} R_{H(S+V)}^{\text{ISR}}(z) \otimes d\hat{\sigma}(z) &= \left(\frac{\alpha}{2\pi}\right)^2 LP_\Theta^{(0)} \otimes R_H^{\text{ISR}} \otimes d\sigma^{(0)} \\ &\quad - \left(\frac{\alpha}{2\pi}\right)^2 LP_\Theta^{(0)} \otimes (LP_\Theta^{(0)} + d_\Theta^{(1)}) \otimes d\sigma^{(0)} - \left(\frac{\alpha}{2\pi}\right)^2 R_{HH}^{\text{ISR}} \otimes d\sigma^{(0)} \\ &\quad + \frac{\alpha}{2\pi}LP_\Theta^{(0)} \otimes \left(d\sigma_{\text{Soft}}^{(1)} + d\sigma_{\text{Virt}}^{(1)} - \frac{\alpha}{2\pi}(LP_\Delta^{(0)} + d_\Delta^{(1)})d\sigma^{(0)}\right) \\ &\quad + \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{L^2}{2}P^{(0)} \otimes P^{(0)} + LP^{(0)} \otimes d^{(1)} + LP^{(1,\gamma)\text{str}}\right)_\Theta \otimes d\sigma^{(0)}. \end{aligned} \quad (16)$$

Using the tables of convolution integrals [11] we obtain the answer for the ISR factor

$$\begin{aligned} R_{H(S+V)}^{\text{ISR}}(z) \otimes d\hat{\sigma}(z) &= \delta_{(S+V)}^{(1)} R_H^{\text{ISR}}(z) \otimes d\sigma^{(0)}(z) \\ &\quad + \left(\frac{\alpha}{2\pi}\right)^2 L \left[2 \frac{1+z^2}{1-z} \left(\text{Li}_2(1-z) - \ln(1-z) \ln z \right) \right. \\ &\quad \left. - (1+z) \ln^2 z + (1-z) \ln z + z \right] \otimes d\sigma^{(0)}(z), \\ \delta_{(S+V)}^{(1)} &= \frac{d\sigma_{\text{Soft}}^{(1)} + d\sigma_{\text{Virt}}^{(1)}}{d\sigma^{(0)}}. \end{aligned} \quad (17)$$

To get the final state radiation factor we use again the Gribov–Lipatov relation and get

$$\begin{aligned} R_{H(S+V)}^{\text{FSR}}(z) d\hat{\sigma} &= \delta_{(S+V)}^{(1)} R_H^{\text{FSR}}(z) d\sigma^{(0)} \\ &\quad + \left(\frac{\alpha}{2\pi}\right)^2 L \left[\frac{1+z^2}{1-z} (-2\text{Li}_2(1-z) - 3\ln^2 z + 2\ln(1-z) \ln z) \right. \\ &\quad \left. + (1+z) \ln^2 z - (1-z) \ln z - 1 \right] d\sigma^{(0)}. \end{aligned} \quad (18)$$

5 Conclusions

In this way, we received the explicit expressions for the radiation factors, which describe hard collinear photon emission in the second order of the perturbation theory within the next-to-leading logarithmic approximation. These factors are universal. They can be used in analytic and numeric calculations of QED radiative corrections to a wide range of processes. In particular, we are going to implement them into the Monte

Carlo event generators LABSMC [12], SAMBHA [13], and MCGPJ [14] for several high energy processes. Our results can be exploited also to provide advanced theoretical predictions for experimental observables with so-called tagged photons, when hard photons emitted at zero (small) angles with respect to colliding charged particles are detected [15,16,17].

Acknowledgements

We are grateful to E. Kuraev for discussions. This work was supported by the RFBR grant 07-02-00932 and the INTAS grant 05-1000008-8328. One of us (A.A.) thanks also the grant of the President RF (Scientific Schools 5332.2006).

Appendix. Explicit Formulae for QED Splitting Functions

The QED splitting functions corresponding to photonic corrections in the leading logarithmic approximation in the first and second orders read

$$\begin{aligned}
P_{ee}^{(0)}(z) &= \left[\frac{1+z^2}{1-z} \right]_+ = \lim_{\Delta \rightarrow 0} \left\{ \delta(1-z) P_{\Delta}^{(0)} + \Theta(1-z-\Delta) P_{\Theta}^{(0)}(z) \right\}, \\
P_{\Delta}^{(0)} &= 2 \ln \Delta + \frac{3}{2}, \quad P_{\Theta}^{(0)}(z) = \frac{1+z^2}{1-z}. \\
P_{ee}^{(0)} \otimes P_{ee}^{(0)}(z) &= \lim_{\Delta \rightarrow 0} \left\{ \delta(1-z) \left[\left(2 \ln \Delta + \frac{3}{2} \right)^2 - 4\zeta(2) \right] \right. \\
&\quad \left. + \Theta(1-z-\Delta) 2 \left[\frac{1+z^2}{1-z} \left(2 \ln(1-z) - \ln z + \frac{3}{2} \right) + \frac{1+z}{2} \ln z - 1 + z \right] \right\},
\end{aligned} \tag{A.1}$$

where symbols δ and Θ denote the Dirac δ -function and the step function, respectively. The space-like (ISR) and time-like (FSR) next-to-leading terms of the QED splitting functions for photonic corrections can be cast as [5,8]

$$\begin{aligned}
P_{ee}^{(1,\gamma)\text{str}}(z) &= \delta(1-z) \left(\frac{3}{8} - 3\zeta_2 + 6\zeta_3 \right) + \frac{1+z^2}{1-z} \left(-2 \ln z \ln(1-z) \right. \\
&\quad \left. + \ln^2 z + 2\text{Li}_2(1-z) \right) - \frac{1}{2} (1+z) \ln^2 z + 2 \ln z - 2z + 3, \\
P_{ee}^{(1,\gamma)\text{frag}}(z) &= \delta(1-z) \left(\frac{3}{8} - 3\zeta_2 + 6\zeta_3 \right) + \frac{1+z^2}{1-z} \left(2 \ln z \ln(1-z) \right. \\
&\quad \left. - 2 \ln^2 z - 2\text{Li}_2(1-z) \right) + \frac{1}{2} (1+z) \ln^2 z + 2z \ln z - 3z + 2.
\end{aligned} \tag{A.2}$$

In the next-to-leading calculations we need also the initial condition for the structure and fragmentation functions at a certain scale μ_0 :

$$d^{(1)}(z, \mu_0, m_e) \equiv d^{(1)}(z) = \left[\frac{1+z^2}{1-z} \left(\ln \frac{\mu_0^2}{m_e^2} - 2 \ln(1-z) - 1 \right) \right]_+. \tag{A.3}$$

The dilogarithm and the Riemann zeta-function are defined as usually:

$$\text{Li}_2(x) = \int_0^1 dy \frac{\ln(1-xy)}{y}, \quad \zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}. \quad (\text{A.4})$$

References

- [1] A.B. Arbuzov, G.V. Fedotovitch, E.A. Kuraev, N.P. Merenkov, V.D. Rushai, L. Trentadue, JHEP **10** (1997) 001.
- [2] A.B. Arbuzov, V.A. Astakhov, E.A. Kuraev, N.P. Merenkov, L. Trentadue, E.V. Zemlyanaya, Nucl. Phys. B **483** (1997) 83.
- [3] G. 't Hooft and M.J.G. Veltman, Nucl. Phys. B **153** (1979) 365.
- [4] E.A. Kuraev and V.S. Fadin, Sov. J. Nucl. Phys. **41** (1985) 466.
- [5] F.A. Berends, W.L. van Neerven and G.J.H. Burgers, Nucl. Phys. B **297** (1988) 429 [Erratum-ibid. B **304** (1988) 921].
- [6] M. Skrzypek, Acta Phys. Polon. B **23** (1992) 135.
- [7] A.B. Arbuzov, Phys. Lett. B **470** (1999) 252.
- [8] A. Arbuzov, K. Melnikov, Phys. Rev. D **66** (2002) 093003.
- [9] A.B. Arbuzov and E.S. Scherbakova, JETP Lett. **83** (2006) 427.
- [10] A.B. Arbuzov, E.A. Kuraev, N.P. Merenkov and L. Trentadue, Nucl. Phys. B **474** (1996) 271.
- [11] A.B. Arbuzov, *Tables of convolution integrals*, hep-ph/0304063.
- [12] A.B. Arbuzov, *LABSMC: Monte Carlo event generator for large-angle Bhabha scattering*, hep-ph/9907298.
- [13] A.B. Arbuzov, D. Haidt, C. Matteuzzi, M. Paganoni and L. Trentadue, Eur. Phys. J. C **34** (2004) 267.
- [14] A.B. Arbuzov, G.V. Fedotovitch, F.V. Ignatov, E.A. Kuraev and A.L. Sibidanov, Eur. Phys. J. C **46** (2006) 689.
- [15] M.W. Krasny, W. Placzek and H. Spiesberger, Z. Phys. C **53** (1992) 687.
- [16] A.B. Arbuzov, E.A. Kuraev, N.P. Merenkov and L. Trentadue, JHEP **9812** (1998) 009.
- [17] H. Anlauf, A.B. Arbuzov, E.A. Kuraev and N.P. Merenkov, JHEP **9810** (1998) 013.